Notes on the Economics of Redevelopment by Demolition

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Abstract. In the present paper I explore the issue of redevelopment by demolition. For that I invoke a Cobb-Douglas model of floor-space production. And within that framework find an optimal (non-constrained) allocation of the inputs land and capital. That leads directly to a well known real estate concept, that of optimal floor-area ratio (FAR). Such an optimal FAR is associated with the maximization of land value. To complete the exposition, non-optimal land value and FAR are derived to represent a situation where public plans constrain FAR.

Housing production, optimal FAR and Land Value

Many authors, even in recent papers, equate housing consumption with the amount of land a household occupies, but that approach ignores the important role that land-capital trade-off plays in the production of buildings, and therefore also in the determination of optimal floor-area ratio (FAR) and the interplay between floor-area value and urban land value. So here, I assume (as in Muth (1970), Richardson (1983), Hatta and Ohkawara (1993), Bertaud and Bruekner (2003) and others) a Cobb-Douglas building production function (in other words, the construction function)

(1) $$F = AL^{\beta}C^{1-\beta}$$

Where F stands for floor area produced

L is the amount of land input and λ is the price of land

C is the capital input and k is its rental price

β is the weight of land in housing production

A is the productivity.

Assuming monopolistic competition, each producer maximizes profits Π, but the best he can do is to break even. For now, the duration of construction is ignored, and with it, the intermediate financing of construction. Thus, the price of capital is 1 rather than 1+r; the important aspect of construction duration will be added to the model further on. The price of land is λ and the price of floor-area is V.

(2) $$\max \Pi = V \cdot F - \left[\lambda L + C\right] = V \cdot A \cdot L^{\beta} C^{1-\beta} - \left[\lambda L + C\right]$$

First order conditions for optimality are:

(3) $$\begin{aligned} \frac{\partial \Pi}{\partial L} &= \frac{\beta}{L} F \cdot V - \lambda = 0 \\ \frac{\partial \Pi}{\partial C} &= \frac{1-\beta}{C} F \cdot V - 1 = 0 \end{aligned}$$

Dividing we get

(4) $$\lambda^{*} = \frac{\beta}{1-\beta} \cdot \frac{C}{L}$$

From (3.6)

(5) $$\frac{C}{L} = \left[\frac{FAR}{A}\right]^{\frac{1}{1-\beta}}$$

Combining (3.9) and (3.10)

(6) $$\lambda^{*} = \frac{\beta}{1-\beta} \cdot \left[\frac{FAR}{A}\right]^{\frac{1}{1-\beta}}$$

From (3.a)

(7) $$FAR = \frac{\lambda^{*}}{\beta \cdot V}$$

Combining (6) and (7) and simplifying results in the following relationships between housing land value λ, housing floor-area value V and the optimal floor-area ratio FAR:

(8) $$\lambda(x) = \beta (1-\beta)^{\frac{1-\beta}{\beta}} A^{\frac{1}{\beta}} V^{\frac{1}{\beta}}(x)$$

(9)

$$FAR = A^{\frac{1}{\beta}}(1-\beta)^{\frac{1-\beta}{\beta}} V^{\frac{1-\beta}{\beta}}$$

Discussion: Equation (8) explicitly states an important attribute of the real estate market: lambda, the value of land (a critical input) is highly sensitive to V the value of floor area (the output). In fact, measured beta values (in Israel) are close to .25 or .3, lambda therefore grows (or declines) at the 3rd or even 4th power of V. This is not just an economic curiosity. It means that purchasing land is identical to buying a highly leveraged derivative claim on an underlying asset – the potential for developing the building.

If the rental market for floor area goes up, or is pushed up by various strategies (basically, by "creating a new and desirable address") then there is an opportunity for a huge capital gain. When it goes down, the potential for major capital loss is there too. This attribute of the real estate market has created and destroyed large fortunes. It is also completely overlooked by some politicians who recommend de-privatizing land (the Lloyd George school) – thus destroying the major incentive for innovative real estate ventures.

In Israel, it is known that private housing prices are relatively stable, while the traded security prices of real estate firms (whose main product is housing) are highly volatile. A possible explanation may be volatile land inventories of these firms.

Time to Market and the Cost of Capital: Land Value when putting up a building takes time

The derivation of land value is repeated here under the assumption that construction takes time and therefore needs interim financing. The simple "instant duration / no financing" case is then a special case with zero values for T and k.

Instead of maximizing income (as in the static model), the real estate builder needs to maximize the discounted cash flow for creating and selling the project. According to the revised model, the best result he can get is to earn an annual return of k on capital, the going rate of return for the type of development undertaken and its associated risk class. Whatever extra profit is to be had is converted to a higher land value.

Cash flow is made up of three parts: (1) future revenue equal to the expected value of the produced floor area; (2) a gradually paid out cost of construction (T installments of size C/T); and (3) $\lambda \cdot$ L the immediate investment of the site's value. Therefore, the term to be maximized is:

$$\Pi = e^{-kT} V\, F - \left[\lambda L + \frac{(1-e^{-kT})}{k\,T} C \right]$$

$$= e^{-kT} V \cdot A \cdot L^{\beta} C^{1-\beta} - \left[\lambda L + \frac{(1-e^{-kT})}{k\,T} C \right]$$

(10)

First order conditions for a maximum are:

$$(a)... \qquad \frac{\partial \Pi}{\partial L} = \frac{\beta}{L} e^{-kT} V\, F - \lambda = 0$$

$$(b)... \qquad \frac{\partial \Pi}{\partial C} = \frac{(1-\beta)}{C} e^{-kT} VF - \frac{(1-e^{-kT})}{k\,T} = 0$$

(11)

Dividing the two equations:

$$\frac{\frac{\beta}{L} e^{-kT} V\, F}{\frac{(1-\beta)}{C} e^{-kT} VF} = \frac{\lambda}{\frac{(1-e^{-kT})}{k\,T}}$$

(12)

Isolating λ:

(13) $$\lambda = \frac{(1-e^{-kT})}{kT} \cdot \frac{\beta}{(1-\beta)} \cdot \frac{C}{L}.$$

Plugging $F = AL^{\beta}C^{1-\beta}$ into (11a) we get

(14) $$\lambda = \frac{\beta}{L} e^{-kT} V \cdot A \cdot L^{\beta} C^{1-\beta} = \beta e^{-kT} VA \left[\frac{C}{L}\right]^{1-\beta}$$

Now combine equations (13) and (14) to get an expression for the price of land λ in terms of V, the price of built up floor area:

(15) $$\lambda = \beta (1-\beta)^{\frac{1-\beta}{\beta}} \left[\frac{kT}{1-e^{-kT}}\right]^{\frac{1-\beta}{\beta}} \left[e^{-kT} AV\right]^{\frac{1}{\beta}}$$

To check the consistency of this result we revert to the instant building time: On setting k and T to zero we get the special result

(16) $$\lambda = \beta (1-\beta)^{\frac{1-\beta}{\beta}} \left[AV\right]^{\frac{1}{\beta}}$$

which is equivalent to equation (8) above. In what follows expression (16) is used to keep the math un-cluttered.

Redevelopment by Demolition

New construction sometimes takes place at inner city sites and not just at the city edge. (Some of these inner sites are remnants of past industrial or junk-yard land-uses and may have been contaminated and are in need of de-contamination; they are called brown-fields and are not treated here.)

To understand redevelopment by demolition in the open market, let us follow the main procedure: we take an active building that is rentable (and therefore has a positive market value) and we destroy it. In return, we get a build-able site – just land. Therefore, the first and foremost question to ask is: will we get a site whose value (land alone) is greater than the value of the demolished asset (including the cost of demolition)? And if so, is it sufficiently more valuable so as to equal or exceed alternative investments of similar risk level? The formal way of writing this is:

(17) $$\frac{\lambda_{after} \cdot \underline{L} - (V_{before} + \delta) \cdot \underline{F}}{(V_{before} + \delta) \cdot \underline{F}} = \frac{\lambda_{after} \cdot \underline{L}}{(V_{before} + \delta) \cdot \underline{F}} - 1 \geq (1 + \tilde{\rho})^T - 1$$

Here $\delta > 0$ is the cost of demolishing one sqm of the old building, T is the expected duration of the demolition and site preparation (not the total project duration), and $\tilde{\rho}$ is the expected rate of return on an alternative investment, adjusted for risk.

This necessary condition yields an important result regarding the desired FAR of the planned new building: Assume that the before-value of a built sqm is equal to its after-value multiplied by a discount factor $0 < \theta < 1$ (due to partial obsolescence) and that the interval between start of demolition and build-able site is a year, then (4.15) is equivalent to

(18) $$\frac{\lambda_{after} \cdot \underline{L}}{V_{before} \cdot \underline{F}} > \frac{\lambda_{after} \cdot \underline{L}}{(V_{before} + \delta) \cdot \underline{F}} \geq 1 + \tilde{\rho} \quad \Rightarrow \quad \frac{\lambda_{after}}{V_{after}} \geq \theta \cdot (1 + \tilde{\rho}) \frac{\underline{F}}{\underline{L}}$$

However, because of optimality

(19) $$\frac{\lambda_{after}}{V_{after}} = \beta \frac{F_{after}}{\underline{L}} = \beta \cdot FAR_{after} .$$

The last two expressions yield the result that the optimal new FAR must satisfy

(20) $$FAR_{after} \geq \frac{\theta}{\beta} \cdot \left(1+\tilde{\rho}\right) \cdot FAR_{before} \ .$$

In other words, in order to achieve market feasibility, condition (17) must hold. If in fact that condition holds, then it implies condition (20) which dictates the newly developed building will need to have a optimal new-FAR that is at least $\frac{\theta}{\beta} \cdot \left(1+\tilde{\rho}\right)$ times the old-FAR. In cases where the existing building is still in good condition, θ is close to one. Beta is usually between 1/4 and 1/3, and the required cost of capital 10% or more. Thus we are talking at least a 3 to 4 fold increase in FAR that is consistent with the projected new land value. If for some reason (e.g. zoning, neighborhood protest, terrain instability) the optimal FAR cannot be achieved, then the projected land price λ_{after} needs to be revised downwards. This is the subject of the next section.

Land Value When FAR is Non-Optimally Constrained

The optimal allocation divides up the value of a finished unit of floor-space into the contribution of the inputs to construction: land and non-land, with no "excess profit" remaining. Due to competition, the "no excess profit" condition remains always; and this result is not limited to optimality. Therefore, given a "dictated" FAR and V, the market value of floor-space, λ, the reduced value of land is the residual left after subtracting the value of non-land inputs from the value of floor-space. Thus:

(21) $$\lambda = V \cdot FAR - \frac{C}{L}$$

From the housing production function (which always holds as well) we get:

(22) $$F = A \cdot L^{\beta} C^{1-\beta} \quad \Rightarrow \quad \frac{C}{L} = \left(\frac{FAR}{A}\right)^{\frac{1}{1-\beta}}$$

Combining (216) and (22) we get

(23) $$\lambda(FAR) = V \cdot FAR - \left(\frac{FAR}{A}\right)^{\frac{1}{1-\beta}} .$$

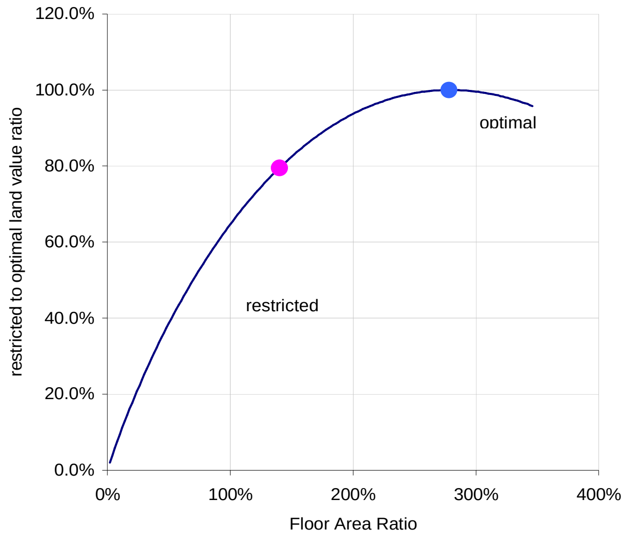
Expression (23) is always true; the optimum land price λ^* is its special case when optimal FAR* is substituted.

Exhibit (1) plots λ/λ^* as the function of FAR satisfied by eq. (23). (FAR) is everywhere concave and thus its slope is very shallow on both sides of the maximum. This means that even significant deviations from the optimal FAR can be made without giving up too much land value. The reason is that while real estate value is lost, non-land inputs are being saved at an increasing rate (increasing marginal costs in reverse). In the example shown, (exhibit 1), while optimal FAR Is 280%, allowing only half of it, 140%, as restricted FAR reduces land value by 20% only. Developers should keep that in mind when negotiating with city-hall for that last bit of FAR.

It should be noted that although FAR zoning restrictions reduce the value of the sites that are so restricted, the macro effect of a severe FAR restrictions policy is to reduce the city's supply of real-estate floor area, thus leading to an overall rise in real-estate prices. (See "Why is Manhattan so expensive", Glaeser et al).

The arguments in this section have been developed with housing in mind. However, they also apply to non-housing land uses, e.g. to office and other high-rise commercial development.

Exhibit 1: Non-Optimal to Optimal Land-Value Ratio as a function of FAR



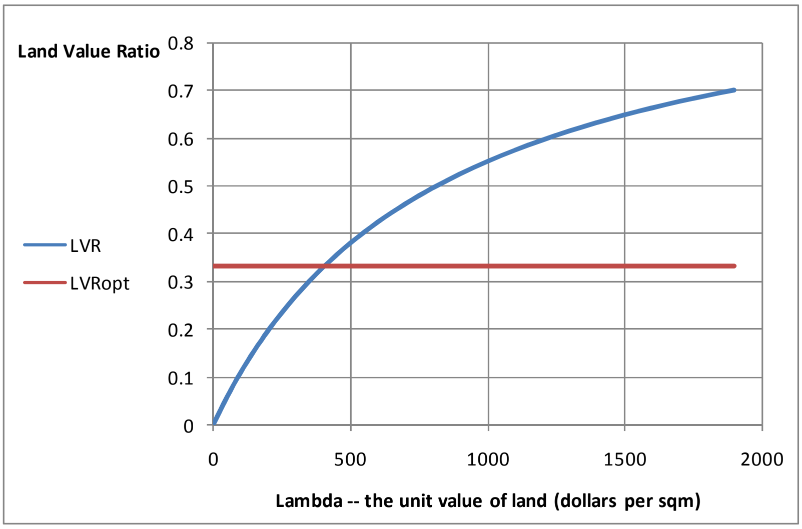
Variable Land Value Ratio when FAR is Constrained. Define land value ratio (LVR) to be the ratio of land value to total project value. Under Cobb Douglas and optimality LVR is equal to beta; in other words it is a constant between zero and one. However, when FAR is constrained below the optimal FAR, a constant LVR is no longer the case. LVR then becomes a function of land value λ. Theoretically, it can take up a range of values between zero and one. Here is why: Competition between builders (still) drives excess profits to zero. Thus land value and capital exactly add up to the total value. Based on 22 and 23,

(24) $$V \cdot F = \lambda L + C$$

(25) $$LVR(\lambda) = \frac{\lambda L}{\lambda L + C} = \frac{\lambda L}{\lambda L + \left(\frac{\overline{FAR}}{A}\right)^{\frac{1}{1-\beta}} \cdot L} = \frac{\lambda}{\lambda + \left(\frac{\overline{FAR}}{A}\right)^{\frac{1}{1-\beta}}}.$$

Since FAR is not allowed to vary and take on the optimal value, the LVR function become increasing in lambda; it starts at the origin and is bounded from above by one.

Exhibit 2: Land Value Ratio under optimal and exogenous FAR



Conclusions and Future Research

So far, there has not been a single empiric hint that the floor-area construction function follows the Cobb-Douglas specification. However, as in numerous theoretical presentations, here too it is highly useful to make the Cobb-Douglas assumption due to the richness of results that it yields.

In the present paper I explored the important issue of redevelopment by demolition. To do that it was necessary to invoke a somewhat known model of floor-space production. The procedure began by assuming an optimal (non-constrained) allocation of the inputs, land and capital. That led directly to a well known real estate concept, that of optimal floor-area ratio (FAR). Such an optimal FAR is associated with the maximal land value. To complete the exposition, non-optimal land value and FAR were derived to represent a situation wherein plans constrain FAR.

Two further issues might be researched so as to immediately further this direction of inquiry: (1) estimating and validating the floor-area production function; and (2) empiric research of successful development by demolition projects.

References

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