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On the optimal spatial structure of property tax[1]

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Abstract

The property tax on residential properties in an urban setting is distortive in two distinct dimensions. The first, inherent in commodity taxation, is the divergence of MRS from MRT; the second, which is relevant to spatial setting, is the uniform application of the tax over space, given the above divergence. The prevailing literature on property tax is almost entirely concerned with the first dimension. The study reported here endeavors to fill this gap. First, we define the issue in a setup of a closed urban system, consisting of open monocentric cities; second, we characterize the determinants of the (second-best) optimal spatial tax structure; and third, we explore its relationship to the controversial reform designed to increase the tax rate falling on land above the tax rate falling on the improvement (split-rate tax structure). We show that if, initially, the same tax rate that applies equally to land and improvement varies optimally across locations, introducing a split-rate structure is always welfare improving. We can not show, however, that, this is also true if initially the common tax rate does not vary optimally across locations. The reason is that if one requirement for efficiency is violated, then pursuing the satisfaction of any of the other requirements need not be welfare increasing.

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1. Introduction

This article is concerned with the (second-best) optimal spatial structure of the tax on residential real property value in a closed urban system composed of open monocentric cities. We consider this issue important in itself and in its implications on the disputed tax reform occupying the recent literature on property tax. The present paper characterizes the spatial (second best) efficient tax rate distribution and discusses the implication of overlooking this aspect of the tax in evaluating the merit of the reform discussed in the literature.

Property tax on real residential property itself is a generic term of a host of tax systems differing from each other by the specific tax base or bases, their assessment in theory and practice, the level of government that determines rules for the above characteristics, the level of governments that collect and use the revenue of these taxes, etc. (see Youngman and Malme (1994)). Here we suggest a prototype, often used in the literature, to portray a simplified version of the prevailing property tax systems. According to this prototype, a fixed ad-valorem tax is levied on the value of each residential property consisting of a land parcel occupied by residential structure. Such a tax is equivalent to another ad-valorem tax system imposed separately, but at the same rate, on the land parcel and the structure (improvement) of the property (e.g., see Brueckner and Kim (2003)).[4]

Most of the recent studies on property tax are concerned with a reform designed to replace the existing uniform rate with a split-rate (or graded) property tax such that the rate on land exceeds the rate on the improvement.[5] This reform is motivated by the prevailing view that taxing land that is immobile is not distortive whereas the tax falling on the improvement is distortive for two reasons. First, the portion of the tax falling on the improvement is a commodity

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[4] An important simplification inherent in their formulation that we follow is the abstraction from the multi period dimension inherent in the concept of value. We follow them in using a single period horizon model.

[5] Such a split-rate system prevails in 18 cities in Pennsylvania, including Pittsburgh., as well as other local governments and municipalities elsewhere (e.g., South Korea and Australia - see Youngman and Malme (1994)).

tax that creates a distortive gap between MRS and MRT. Second, in a spatial context, it is often alleged that, by reducing the structural density, the property tax falling on the improvement tends to induce sprawl which is often considered to be harmful.

The specific controversial issues regarding the merits of this reform are whether, indeed, land tax in practice is not distortive and whether the tax on the improvement that is inherent in the property tax really induces sprawl (a concept which is not always well defined). The first issue has originally been raised independently of the property tax reform's controversy (e.g., see Bentick (1979), D. E. Mills (1981), Wildasin (1982), and Tideman (1982)) but it is directly relevant to the potential merit of the reform. It turns out that the non-neutrality of land taxation finding is based on an assumed assessment practice where the site valuation depends on the value of the existing improvement instead of the value of raw land which reflects the best use of the site (see Wildasin (1982), Tideman (1982), and Arnott and Petrova (2002)). This solution to the conceptual debate, however, is extremely difficult to implement. Arnott and Petrova (2002) describe the issue as a dilemma between "greater efficiency of the raw site value tax and the lower administrative costs (broadly speaking) of the residual tax". E. S. Mills (1998) is more skeptical regarding the reform when he argues that even residual assessment (e.g., assessment based on the property value minus the cost of the improvement) is vulnerable to substantial mistakes, especially when it applies to old buildings that constitute a considerable fraction of the city cores' residential properties. E. S. Mills (1998) suspects that due to these practical difficulties, the reform may generate more distortion than generated by the *existing* property tax and concludes that "...a land tax, substantially substituted for the existing property tax, is theoretically attractive but practically almost worthless."

The second issue (the effect of property tax on sprawl) was investigated on both theoretical and empirical levels. On the first level, Brueckner and Kim (2003) showed that the property tax has two opposing effects on urban sprawl (when we define it as the average urban population density). On the one hand, the tax on the improvement tends to reduce structural density whereas, on the other hand, the substitution and welfare effects reduce the housing consumption. Therefore, the combined effect on population density and sprawl is ambiguous, depending on the relevant functions and their parameters.[6]

On the empirical level, the issue of the effect of property tax on sprawl was studied either indirectly, through the effect of applying the split-rate reform (e.g., see Oates and Schwab (1997)), or directly by correlating city area size to the effective tax rate (e.g., see Song and Zenou (2006)). Oates and Schwab (1997) studied the implications of the property tax reform carried out in Pittsburgh in 1979-80 (under this reform, the tax rate on land was raised to more than five times the tax rate on structures). They concluded that this reform played an important supporting role in "…the dramatic increase in the building activities, far in excess of the other cities in the region". Their result refers mainly to structural density of office buildings and can hardly support the presumption that the reform can restrain sprawl. A recent study by Banzhaf and Lavery (2007), however, provides more relevant evidence that the combined effects of the split-rate tax scheme on population density is positive. On the one hand, they show that the split-rate tax scheme increased the capital/land ratio substantially; on the other hand, not ignoring the opposing effect of the property tax emphasized by Brueckner and Kim (2003), they found that its effect on the per-capita housing demand was insignificant. They conclude, therefore, that "…the split-rate tax is potentially a powerful anti-sprawl tool."

The reported empirical finding, however, is not conclusive. Song and Zenou (2006) find a negative correlation between the census urbanized areas, on the one hand, and the average property tax, on the other, thus suggesting that the dwelling size effect of Brueckner (2003) dominates the structural density effect.

Although the prevailing property tax rate is not always uniform across space (see Youngman and Malme (1994)), this is not reflected in the theoretical literature that we are aware of; rather, the property tax is portrayed as an ad-

[6] Earlier studies either were not aware of these two opposing effects (e.g., see Nechyba (1998)) or were aware of both but were not concerned with the effect of the property tax on sprawl in terms of urban area expansion (e.g., see Leroy (1976)).

valorem tax applied uniformly over space inside the local jurisdictions that employ the tax. Such uniformity violates the (second-best) efficiency of commodity taxation and is an additional source of distortion in property tax systems. This gap in the existing theoretical analysis of the distortion generated by the property taxation motivates the present study.

In order to explore this issue, we construct a simple general equilibrium model of a closed urban economy with a fixed population and a system of open cities (that is, with costless migration between cities), as in Anas and Pines (2008). Constructing such a model requires, of course, definitions of the centrifugal and centripetal forces that allow the endogenous formation of cities with some finite population mass. These two forces have received in the literature diverse and sometimes even conflicting explanations in the relevant literature (see Abdel-Rahman and Anas (2004) for a survey). For example, the centripetal force in Stiglitz' (1977)[7] LPG model is the advantage of a large city size in reducing the per-capita burden of LPG provision while the centrifugal force is production that exhibits scale diseconomies due to a fixed supply of land. In a typical urban model, the centripetal force is rather associated with production that exhibits scale economies, such as those arising from information exchange among producers in the same city or those arising from the taste for a variety of products to consume, or a technological bias for a variety of inputs in production. The centrifugal force is often depicted as per-capita commuting cost required to accommodate the urban population and which increases with population, given the level of utility (see Fujita (1988)[8]). In order to simplify the analysis in our city system model, we borrow the centripetal force from Stiglitz (1977) and the centrifugal force from Fujita (1988).

Using this urban setup, we investigate the determinants of the optimal (second-best) spatial tax structure and explore its relationship to the controversial reform of a split-rate tax scheme. Specifically, we show that, indeed, when the production function of housing is Cobb-Douglas and the compensated demand

[7] See also Atkinson and Siglitz (1980).
[8] See also Fujita and Thisse (2002).

for housing is unitary, there is no reason to differentiate the property tax rate according to location. This, however, is an exceptional case. In general the optimal tax should vary across locations. For example, if the compensated demand elasticity for housing is larger than 1 whereas the elasticity of substitution between composite good and land in the production of housing is smaller than 1, then the (second-best) optimal property tax rate function is decreasing with distance from the city center.

A more general conclusion is that if the tax rate is spatially (second-best) optimal, then a split-rate reform always increases welfare. When, however, initially the tax rate is spatially uniform, marginal increase in the rate falling on land need not be welfare improving. This can be explained by the second-best theory which states that if one requirement for efficiency is violated, it is not clear that each of the other requirements should still be satisfied. The policy implication is that the split-rate tax reform should be accompanied by adjusting the tax rate spatially.

Section 2 introduces the setup. Section 3 discusses the first-best allocation and the implied Henry George rule. Section 4 discusses the second-best property tax structure obtained by its differentiation across locations. Cases where the tax rate should increase, decrease, or remain constant across locations, are identified. Section 5 analyzes the implications of introducing the split-rate tax structure when its spatial structure is optimal and when it is not. Section 6 summarizes the results derived in the paper and specifies the need for further studies, including simulations to obtain a further insight on the importance of extending the scope of the presently conceived reform in the direction implied by this study.

2. The setup

We consider a closed mixed urban economy where the list of agents includes, farmers, urban land-trading corporations (ULTC, hereinafter), workers-residents, (residents, hereinafter) urban housing producers (UHP, hereinafter), and a planner.

Geography: The geography is composed of a system of m identical continuous monocentric cities surrounded by agriculture production.[9] The shape of each of the cities is portrayed by the length of the arc at distance z from the center available for urban use, $\phi[z]$ (e.g., when the cities are circular, $\phi[z]=2\pi x$; when the cities are linear, $\phi[z]=\text{constant}$). The most distant location where $\phi[z]>0$ is defined as the city boundary, b. The relative location of the city centers is unidentified (the intra-city geography is specified; the inter-city is not). The urban population is composed of N identical workers-residents (referred to hereafter as "residents") who live in m identical cities, such that each city accommodates $n=N/m$ residents. The population spatial distribution in each city is portrayed by $n[z]\equiv d\nu[z]/dz$ where $\nu[z]$ denotes the number of residents living within a distance z from the city center.

Farmers:[10] The marginal land productivity in agricultural is constant r, which is the (perfect elastic) supply price for UHP.

Urban land trading corporations: ULTC is a competitive industry where each firm rents land from a farmer at a cost of r and rents it out to the highest bidder at a price of $R(z)$ such that the gross profit per land unit at z of a *ULTC* is $R[z]-r$. Since entry into the land trade business is costless, there is no opportunity for deriving profit from arbitrage at the city boundary, that is,

$$R[b]-r=0. \tag{1}$$

The net profits derived by all the *ULTC* in the city are given by $\Pi=\int_0^b \phi[z](1-\Omega)(R[z]-r)dx$, where Ω is the tax rate on gross profits from land

[9] We can alternatively assume that the cities are surrounded by raw land which has no alternative use.
[10] If we adopt the alternative geographical configuration, there are no farmers (see footnote 8).

transactions. The net tax profits are equally distributed among the city residents who are the ultimate owners in equal shares of the *ULTC*, such that each resident receives $\pi = \frac{m}{N}\Pi$.

Residents: The urban residents are identical in terms of both preferences and initial endowments. Therefore, in equilibrium, each of the N individuals must be accommodated and become a resident in some location of the urban system. Otherwise, an individual left outside will not be able to achieve the utility, which is achievable by those accommodated into the system. Competition among the equal individuals (who have the same income and preferences) must, therefore, lead to:

$$n - \int_0^b n[z]dz = 0, \qquad (2)$$

and

$$mn - N = 0. \qquad (3)$$

The preferences are represented by a quasi-concave utility function $\hat{u}[x,h]$ where x is the quantity of the composite good and h is the quantity of housing. Free mobility, equal income, and competition imply that the utility achieved by each individual in the urban system is the same, that is, $u = \hat{u}[x[z],h[z]]$.

Given the consumer's price of housing q, the compensated demand for the composite good of a resident living at any z can be written as $x[q[z],u]$ and the compensated demand for housing as $h\left[q[z],u\right]$, such that the budget constraint of each household residing at z is:

$E\left[q[z],u\right]+t[z]\equiv x\left[q[z],u\right]+h\left[q[z],u\right]q[z]+t[z]-y=0$,

where $t[z]$ is the commuting cost incurred by a resident living at z, $t'[z]>0$, $t''[z]>0$.

Each resident is endowed with one unit of a composite good and, as already explained, has an equal share in all the net profits of urban land transactions in the economy. Since, however, in equilibrium the cities are identical, it can also be viewed as if each individual has an equal share only in the profits derived by transactions in his own city of residence. Since the no labor income is π, the net income, y, is given by 1+π, thus implying

$$E\left[q[z],u\right]+t[z]-(1+\pi)=0 \tag{4}$$

Where

$$\pi-\frac{1}{n}\int_{0}^{b}\phi[z](1-\Omega)\left(R[z]-r\right)dx=0 \tag{5}$$

<u>Urban housing producers (UHP)</u>: Each of the *UHP* uses C units of a composite good and L units of land to construct $\bar{H}$ housing units of (e.g., defined in units of floor area) according to a CRS production function, $F(C,L)$. Each unit of a composite good costs $1+\alpha[z]$ at z, where $\alpha[z]$ is the ad-valorem tax rate on the composite good input; each land unit costs $R[z]\left(1+\beta[z]\right)$ at z where $\beta[z]$ is the ad-valorem tax rate on the land input. Due to CRS, we can rewrite the production function in terms of density, that is, $H=f[c]$ where $H=\overline{H}/L$ and $c=C/L$. CRS and profit maximization require that c is a function of $\dfrac{1+\beta[z]}{1+\alpha[z]}R[z]$.

Hence, since H depends on c, it is indirectly a function of $\frac{1+\beta[z]}{1+\alpha[z]}R[z]$. Profit maximizing of the composite good/land ratio requires:

$$qF_L[C,L]=1+\alpha[z]\Rightarrow qH'[c]=qf'\left[c\left[\frac{1+\beta[z]}{1+\alpha[z]}R[z]\right]\right]=1+\alpha[z].$$

Free entry into the industry eliminates profits, thus implying

$$q[z]H[z]-c\left[\frac{1+\beta[z]}{1+\alpha[z]}R[z]\right](1+\alpha[z])-R[z](1+\beta[z])=0 \qquad (6)$$

Clearing markets: There are two markets to be cleared: the composite good market and the housing market. Clearing the composite good market implies:

$$n-\left(\int_0^b\left(n[z]\left(x\left[q[z],u\right]+t[z]\right)+\phi[z]\left(c\left[\frac{1+\beta[z]}{1+\alpha[z]}R[z]\right]+r\right)\right)dz+k\right)=0\ . \qquad (7)$$

Clearing the housing market in each location implies:

$$n[z]h\left[q[z],u\right]-\phi[z]H\left[c\left[\frac{1+\beta[z]}{1+\alpha[z]}R[z]\right]\right]=0\ . \qquad (8)$$

Planner: We assume that the formation of a city requires k units of the composite good. The benevolent urban planner chooses the financial instruments he controls in order to maximize the common utility of the urban population subject to (1) - (8) and his budget constraint

$$k-\int_0^b\left(\Omega\phi[z]\left(R[z]-r\right)+\phi[z]\left(c\left[R(z)\right]\alpha[z]+R[z]\beta[z]\right)\right)dz=0. \qquad (9)$$

Summing up, given a policy set $\{\Omega, \alpha[z], \beta[z]\}$, an equilibrium consists of three functions $\{n[z], q[z], R[z]\}$ and five parameters, $\{u, n, m, b, \pi\}$) which satisfy (1) - (9). It is easy, to show, however, that only eight out of the above nine equations are independent - a reflection of the Walras Law.[11] For example, (9) can be derived from (1), (2) and (4) - (8).[12] We are then left with eight equations (1) - (8) to solve the three equations $\{n[z], q[z], R[z]\}$ and the five parameters $\{u, n, m, b, \pi\}$. Moreover, we can simplify the solution procedure by dividing it into two sequential steps. In the first, we ignore (3) and use the remaining seven equations (1), (2), and (4) - (8) to solve the three functions $\{n[z], q[z], R[z]\}$ and the four parameters $\{u, n, b, \pi\}$. Then, in the second step, we substitute the solution for n obtained in the first step into (6) to solve for m. This is possible because m appears only in (3).

In the sequel, we discuss three regimes. Under the first regime, which we define as the first best, the planner is free to use Ω at any level he prefers. Under the second regime, that is formally rarely used (it is sometimes applied through different assessment procedures) the planner cannot use Ω and cannot apply different rates to the land and the improvement but he can apply different rates across locations:

$$\alpha[z] = \beta[z] = \psi[z] \qquad (10)$$

and $\psi[z]$ is free to vary with z as the planner wishes.

Under the third regime, which we define as a prototype of prevailing property tax schemes, the planner is more constrained. In the most common case, the planner is constrained by

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[11] Walras' Law states that in the set of equations representing all the agents' budget constraints and the market clearing conditions, one of the equations is dependent on the others.

[12] The proof is available from the authors upon request.

$$\alpha[z] = \beta[z] = \bar{\psi} \text{ (constant)}, \tag{11}$$

that is, the planner is constrained to use the same rates on land and the structure and this common rate cannot vary across locations; in the less constrained case, referred to as split-rate property tax, the planner is constrained by

$$\bar{\alpha}(\text{constant}) = \alpha[z] < \beta[z] = \bar{\beta}(\text{constant}), \tag{12}$$

that is, the constraint regarding the tax rate on land is somewhat relaxed, but still remains effective.

3. First best

According to this regime, the planner is free to choose any set of taxes $\{\Omega, \alpha[z], \beta[z]\}$ subject to the independent constraints (1), (2), (4) - (8). In fact, however, the problem can be simplified somewhat. First, we notice that the first order conditions for maximizing utility subject to (1), (2), and (4) - (8) is equivalent to the first-order conditions of maximizing the surplus of the city, that is, the left side of (7), subject to (1), (2), (4)-(6), and (8). Second, we observe that $R[b]$ appears only in (1) and Ω appears only in (5). It then follows from the first-order conditions that their Lagrange multipliers of (1) and (5) vanish and, therefore, we can suppress them in the outset. We are then left with (2) - (4), (6), and (8) as the binding constraints for the first stage (recall that the determination of m is left to the second stage). The corresponding Lagrange function is, therefore:

$$L = n - \int_0^b \left(n[z](x+t) + \phi(c+r)\right)dx - k - \int_0^b \theta\left(E + t - (1+\pi)\right)dx$$
$$- \int_0^b \rho\left(n[z]h - \phi H\right)dx - \int_0^b \lambda\left(qh - (1+\alpha)c - (1+\beta)R\right)dx - \varphi\left(n - \int_0^b n[z]dx\right) \qquad (13)$$

where the following abbreviated notation is used:

$$\left\{x,h,E,c,H\right\} \equiv \left\{x\left[q,u\right],h\left[q,u\right],E\left[q,u\right],c\left[R\right],H\left[c\right]\right\}$$
$$\left\{q,R,t,\phi,\theta,\rho,\lambda\right\} \equiv \left\{q\left[z\right],R\left[z\right],t\left[z\right],\phi\left[z\right],\theta\left[z\right],\rho\left[z\right]\right\}.$$

The first-order conditions with respect to q, R, $n[z]$, n, π, and α are[13]

$$q: \qquad -n[z]\left(x_q + \rho h_q\right) - \theta h - \lambda H = 0, \qquad (14)$$

$$R: \qquad -\phi c'\frac{1+\beta}{1+\alpha} + \rho\phi H'c'\frac{1}{1+\alpha} - \lambda\left(qH'c'\frac{1+\beta}{1+\alpha} - (1+\beta)c' - (1+\beta)\right) = 0 \qquad (15)$$

$$n[z]: \qquad -\left(x + \rho h + t - \varphi\right) = 0, \qquad (16)$$

$$n: \qquad 1 - \varphi = 0, \qquad (17)$$

$$\pi: \qquad \int_0^b \theta dz = 0. \qquad (18)$$

$$\alpha: \qquad \phi c'\frac{1+\beta}{(1+\alpha)^2}R - \rho\phi H'c'\frac{1+\beta}{(1+\alpha)^2}R - \lambda\left(-qH'c'\frac{1+\beta}{(1+\alpha)^2}R + c'\frac{1+\beta}{1+\alpha}R - c\right) = 0 \qquad (19)$$

Add $n(z)\left(x_q + qh_q\right)$ to (14) and rearrange, it will become

[13] We do not need the other first-order conditions to derive our results.

$$n[z](\rho - q)h_q + \theta h + \lambda H = 0, \tag{20}$$

Multiply (15) by $\frac{R}{1+\alpha}$, add the result to (19), and substitute into the result the profit maximizing condition $qH' - (1+\alpha) = 0$ and (6) to obtain:

$$\frac{1}{1+\alpha}\big((1+\alpha)c + (1+\beta)R\big)\lambda = \frac{Hq}{1+\alpha}\lambda = 0 \Rightarrow \lambda = 0\,. \tag{21}$$

It follows from (4), (16), and (17) that

$$\rho - q = -\frac{\pi}{h}\,. \tag{22}$$

Finally, substitute (21) and (22) into (20), divide the result by h, integrate, and use (18) to obtain:

$$-\pi\int_0^b n(z)\frac{h_q}{h}dz + \int_0^b \theta dz = -\pi\int_0^b n(z)\frac{h_q}{h}dz + \mu = 0\,. \tag{23}$$

We assume that h is not infinitely inelastic and, therefore, the left side of (23) is positive. This implies contradiction unless π vanishes. When π vanishes, however, $\Omega = 1$, that is, the entire aggregate differential land rent, $\int_0^b \phi[z]\big(R[z] - r\big)dx$ *(ADLR,* hereinafter*),* is used for financing the LPG. Furthermore, when $\pi = 0$, $E + t = 1$. This implies that *ADLR* is not only entirely used to finance k, but it is the single tax, that is, the single Henry George tax for financing k.

Of course, precisely the same results, including the Henry George rule can more easily be derived from the fundamentals, that is,

minimizing $n-\left(\int_0^b \left(n[z]\left(x[z]+t[z]\right)+\phi\left(c+r\right)\right)dx+k\right)$

subject to

$n[z]h[z]-\phi[z]H[z]=0$; $n-\int_0^b n[z]dz=0$; and $u-u\left(x[z],h[z]\right)$.[14]

Our Henry George result depends crucially on the assumption that cities are perfectly replicable. It need not be satisfied, of course, in a non-replicable closed city. If the closed city happens to be larger than its optimal population size, then, under first best, *ADLR>k* such that the surplus should be redistributed to the residents of as a poll subsidy; if the closed city happens to be smaller than its optimal population size *ADLR<k* and the deficit should be financed by a poll tax.

4. Optimizing the property tax rate across locations

In this section we assume that the planner is constrained by $\Omega=0$ and, in addition, (10) is binding. That is, the same tax rate applies to both the land and the improvement, or, equivalently, the tax applies to the value of the property as a whole. The planner can however determine the spatial distribution of the tax rate $\psi[z]$. Being constrained by (10) the relative cost of land is given by

R $\frac{1+\beta}{1+\alpha}R=\frac{1+\psi}{1+\psi}R=R$ and ψ appears only (6) such that, by the first-order condition, its shadow price vanishes. We can, therefore, suppress it from the Lagrange function. The corresponding Lagrange function then reduces to:

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[14]Observe that, in contrast to the preceding exposition, now $\left\{x[z],h[z],c[z]\right\}$ are quantities, not utility maximizing demands and profit-maximizing composite good input in housing production.

$$L = n - \int_0^0 \left(n[z](x+t) + \phi(c+r) \right) dx - k$$
$$- \int_0^0 \theta\left(E + t - (1+\pi)\right) dx - \mu(\pi - \frac{1}{n}\int_0^0 \phi(R-r) dx - \int_0^0 \rho\left(n[z]h - \phi H\right) dx. \tag{24}$$

The first-order conditions are now

$$q: \qquad -n[z]\left(x_q + \rho h_q\right) - \theta h = 0, \tag{25}$$

$$R: \qquad \frac{\phi}{n}\mu - \phi c' + \rho\phi H' c' = 0 \Rightarrow \frac{1}{n}\mu - c' + \rho H' c' = 0 \tag{26}$$

$$n[z]: \qquad -\left(x + \rho h + t - \varphi\right) = 0, \tag{13}$$[15]

$$n: \qquad 1 - \frac{\pi}{n}\mu - \varphi \quad = \quad 0, \tag{27}$$

$$\pi: \qquad \int_0^b \theta dz - \mu = 0. \tag{28}$$

We will now use these conditions and the constraints to characterize how ψ varies across locations.

Adding $x_q + qh_q = 0$ to (25), it can be rewritten as:

$$n[z]\left(x_q + \rho h_q\right) + \theta h = n[z]\left(\frac{\rho}{q} - 1\right) h_q q + \theta h = 0 \tag{29}$$

Using (4), (13) and (27), we obtain:

[15] We use the original number for an equation that previously appeared.

$$\left(\frac{\rho}{q}-1\right)=-\frac{\pi}{qh}-\frac{\pi}{qh}\frac{\mu}{n}. \tag{30}$$

Substituting (30) into (29) and rearranging yields

$$-n[z]\left(\pi+\frac{\pi}{n}\mu\right)\frac{h_q}{h^2}=-\theta=0\Rightarrow -n[z]\pi\frac{h_q}{h^2}=n[z]\pi\frac{h_q}{nh^2}\mu-\theta=0. \tag{31}$$

Integrating (31) and using (28) yields

$$-\int_0^b n[z]\pi\frac{h_q}{h^2}dz=\mu\int_0^b n[z]\pi\frac{h_q}{nh^2}dz-\int_0^b \theta dz=\frac{\mu}{n}\left(\int_0^b n[z]\pi\frac{h_q}{h^2}dz-n\right). \tag{32}$$

Now, since $h_q<0$,

$$0>\frac{-\int_0^b n[z]\pi\frac{h_q}{h^2}dz}{\int_0^b n[z]\pi\frac{h_q}{h^2}dz-n}>-1.$$

Hence,

$$0>\frac{\mu}{n}>-1. \tag{33}$$

It then follows from (30) and (33) that

$$\rho-q=-\frac{\pi}{h}\left(1+\frac{\mu}{n}\right)<0, \tag{34}$$

that is, the market value of housing is excessive.

Using the first order condition for maximizing the structure density, that is, $qH'-(1+\psi)$=0, (26) can be rewritten as:

$$\begin{aligned}&\frac{\mu}{n}-c'+\rho H'c'=c'\left(\frac{\rho}{q}(1+\psi)-(1+\psi)+\psi\right)+\frac{\mu}{n}\\&=(1+\psi)\left(\frac{\rho}{q}-1\right)c'+\psi c'+\frac{\mu}{n}=0.\end{aligned} \tag{35}$$

Then, substituting (30) into (35), we obtain,

$$\frac{\mu}{n}+c'\psi-c'(1+\psi)\frac{\pi}{qh}\left(1+\frac{\mu}{n}\right)=0. \tag{36}$$

Now, we substitute (10) into (6), differentiate the result with respect to z, use the maximizing profit condition with respect to c, and obtain:

$$(c+R)\dot{\psi}+(1+\psi)R\frac{\dot{R}}{R}-Hq\frac{\dot{q}}{q}=0\Rightarrow\frac{\dot{R}}{R}=\frac{Hq}{(1+\psi)R}\frac{\dot{q}}{q}-\frac{c+R}{(1+\psi)R}\dot{\psi}\ . \tag{37}$$

where a dot denotes a derivative with respect to z, that is, the distance from the center. Next, we differentiate (36) with respect to z and obtain

$$\begin{aligned}&\left(1-\frac{\pi}{qh}\left(1+\frac{\mu}{n}\right)\right)\dot{\psi}\\&=-\left(\psi-(1+\psi)\frac{\pi}{qh}\left(1+\frac{\mu}{n}\right)\right)(\sigma-1)\frac{\dot{R}}{R}-(1+\psi)\left(1+\frac{\mu}{n}\right)\frac{\pi}{qh}h(1-\eta)\frac{\dot{q}}{q}\end{aligned} \tag{38}$$

Combining (37) and (38) we get rid of $\dot{R}$ and:

$$\dot{\psi}=\frac{B}{A}\frac{\dot{q}}{q} \tag{39}$$

where:

$\sigma\equiv\frac{c''R}{c'}$, that is, the local elasticity of substituting C for L in housing production,

$\eta\equiv-\frac{h_q q}{h}$, that is, the compensated demand elasticity for housing with respect to its price q,

$$A = 1 - \frac{\pi}{qh}\left(1+\frac{\mu}{n}\right) - \frac{c+R}{(1+\psi)R}\left(\psi - (1+\psi)\frac{\pi}{qh}\left(1+\frac{\mu}{n}\right)\right)(\sigma-1),$$

and

$$B = -\left(\frac{Hq}{(1+\psi)R}\left(\psi - (1+\psi)\frac{\pi}{qh}\left(1+\frac{\mu}{n}\right)\right)(\sigma-1) + (1+\psi)\frac{\pi}{qh}\left(1+\frac{\mu}{n}\right)(1-\eta)\right).$$

With the definitions of σ, η, A, and B, (39) can be used to characterize $\dot{\psi}$. In some specific cases, we can characterize it analytically. To that end, we first use (33) and (36) to verify that

$$\psi - (1+\psi)\frac{\pi}{qh}\left(1+\frac{\mu}{n}\right) > 0 \Rightarrow \frac{\pi}{qh}\left(1+\frac{\mu}{n}\right) < \frac{\psi}{1+\psi} < 1$$
$$\Rightarrow 1 - \frac{\pi}{qh}\left(1+\frac{\mu}{n}\right) > 0. \quad (40)$$

It follows from (40) and $\dot{q} < 0$ that:

Proposition 1: (a) *If* $\eta = \sigma = 1$*, then* $A > 0,\ B = 0$*, and* $\dot{\psi} = 0$*. (b) If* $\eta > 1$ *and* $\sigma < 1$*, then* $A > 0,\ B > 0$*, and* $\dot{\psi} < 0$*. (c) If* $\eta < 1$*, and,* $\sigma = 1$*, then* $A > 0,\ B < 0$*, and* $\dot{\psi} > 0$*.*

Figure 1 portrays the three cases referred to in Proposition 1 and represents those few cases that we could characterize ψ analytically. The single point in the illustration that generates $\dot{\psi} = 0$ is $(\eta, \sigma) = (1,1)$. Corresponding to the shaded area, including its boundaries $\dot{\psi} < 0$; corresponding to any point on the bold vertical line $(\eta, \sigma) \in [(1,0),(1,1))$ $\dot{\psi} > 0$.

Proposition 1 shows that ψ may increase, decrease, or remain constant, with distance from the city center, where, however, the latter case is less probable than the others. We cannot prove that ψ always changes monotonically with distance from the city center.

Two comments are in order. First, observe that the sufficient conditions in the premises of Proposition 1 are not restricted to constant elasticities, rather the elasticities are defined locally. Finally, observe that a unitary compensated demand elasticity of housing consumption and unitary elasticity of substitution in housing production play an important role in urban economics. They are used by Muth (1961), together with linear transportation cost, to derive the exponential density function.[16]

We now turn to a policy issue associated with the optimal structure of the property tax. Suppose, that the spatial distribution of ψ is efficient, Will a reform designed to increase the rates on land above the rate on structure marginally improve welfare? We will show that the answer is positive. To that end we assume that that initially (10) is satisfied and we increase $\beta\left[z\right]$ at some arbitrary z. the marginal benefit of such change is evaluated by partially differentiating the Lagrangian with respect to $\beta\left[z\right]$ which yields

$$\frac{\partial L}{\partial \beta} = -\phi c'\frac{R}{1+\alpha} + \rho\phi H'c'\frac{R}{1+\alpha} = \phi\frac{R}{1+\psi}\left(\rho H'c' - c'\right). \tag{41}$$

Substituting (26) into (41) and using (33), we obtain:

$$\frac{\partial L}{\partial \beta} = -\phi\frac{R}{1+\psi}\frac{\mu}{n} > 0. \tag{42}$$

It then follows that, given the optimal distribution of $\psi\left[z\right]$, introducing a split-rate property tax system is welfare improving. Is it also true in the common case where (11) applies? This issue is explored in the next section

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[16] Papageorgiou and Pines (1989) provide a general characterization of preferences that generate constant compensated demand elasticity, including the case of unitary elasticity.

5. A prototype of the prevailing property tax and the split- rate tax reform

There are two common features characterizing the prevailing property tax, especially in the U.S. The first is that the tax applies to the value of the residential property as an inseparable bundle. Only in a very few cases, the tax rate on the (assessed) value of land is higher than on the assessed value of the improvement.[17] The second feature is the uniformity of the tax rates across locations. Thus, (11) characterizes the most common property tax scheme where (12) characterizes some very few cases where the split-rate tax scheme is practiced. Implementing the scheme is a controversial issue that occupies the literature on property tax as elaborated upon in the introduction. Here, we will raise another reason for questioning its merit when the tax is uniformly applied over space.

When the tax rate varies across location optimally, (42) represents the marginal benefit of increasing the tax on land locally. We have to integrate it to represent the marginal benefit of raising the tax on land marginally everywhere. When the tax rate does not vary across locations, we obtain similar, but not exactly the same, expression for the marginal benefit of raising the tax rate on land marginally, that is,

$$\frac{\partial L}{\partial \bar{\beta}} = -\frac{\mu}{n\left(1+\bar{\psi}\right)}\int_0^b \phi R dz \tag{43}$$

This difference, however, would not be consequential, if μ remains negative as it is in the preceding section. Unfortunately, we are unable to prove that, when the

[17] Less than twenty cities in Pennsylvania are referred to as practicing the split-rate scheme and a few other places out of the U.S. (see Youngman and Malme (1994)).

tax rate is uniform across locations, $\mu < 0$ as we were able to prove in the preceding section, where the tax rate varies across locations. Formally, the reason is that, in the preceding section $\lambda[z] = 0$ because, for any specific location z, $\psi[z]$ appears only in equation (6) that is relevant to that specific location. In contrast, in the preset case, $\overline{\psi}$ appears in (6) which refers to every location and, therefore, $\lambda[x]$ need not vanish. When $\lambda[x]$ does not vanish, the expression preceding (33) is not valid, and we cannot formally prove (33).

One may be puzzled by this failure to reproduce the result obtained in the preceding section. It is well known and illustrated in Section 3 that, in our setting, Henry George's single tax at a rate of 100% on the *ADLR* is not distortive, that is, it reduces welfare, measured in terms of the composite good, precisely by the same amount as the tax revenue. One, therefore, tends to expect that any reform that increases the property tax rate on land above the rate on the improvement enhances welfare.[18] We suspect that our failure to reproduce the result derived in the preceding section is an illustration of the second-best principle that if one necessary condition for first-best is violated, the satisfaction of all the other conditions need not always be warranted. In this sense our discussion adds a new (theoretical) dimension to the controversy regarding the merit of the split-rate tax scheme.

6. Concluding comments

The resource allocation distortion associated with the residential property tax in an urban setting has two distinct sources. The first is the very use of commodity taxation that generates a gap between MRS and MRT; the second is the uniform application of the tax spatially. The present paper sheds some light on the second source of the above distortion and the characteristics of the

[18] Even E. S. Mills states that the split-rate scheme "…is theoretically attractive " but only "…practically worthless."

optimal spatial structure of the tax rates, given the first distortion. It turns out, that the analysis suggested in this paper is also relevant to the dispute on the merit of the split rate property tax reform, which occupies the recent literature on residential property taxation.

We explore these issues in a general equilibrium setup of a closed urban system of monocentric cities.[19] We derive a condition for determining the (second-best) optimal spatial structure of the tax rates. In some cases we can use identify conditions for determining the spatial rate structure analytically, showing that the tax rate, $\dot{\psi}$, can vanish, be positive or negative globally. Most probably $\dot{\psi}$ need not always be monotone, though we did not identify conditions that generate this possibility.

On a more general level, our discussion adds a new dimension to the controversy on the merit of the split-rate property tax reform. We suggest that when the property tax rate is uniform across location, raising the tax rate on land need not be welfare increasing; only when the spatial tax rate structure is (second-best) optimal, raising the tax rate is. This reflects the well known principle of second best: if one necessary condition for efficiency is violated, efficiency improvement need not follow from pursuing the attainment of another efficiency condition.[20]

Even with all our model's simplifications, it cannot provide a complete characterization of the optimal spatial tax structure analytically. Simulations are still needed to check cases where our formula fails to produce unambiguous results and to gain some notion on the importance of the spatial tax variation in terms of welfare gain.

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[19] By this we mean that all the resources of the urban system are used internally, whereas, in partial equilibrium analysis, resources are allowed to leak out without explaining how they are disposed of.

[20] We did not, however, produce a counter example where the introduction of a split rate tax structure reduces welfare.

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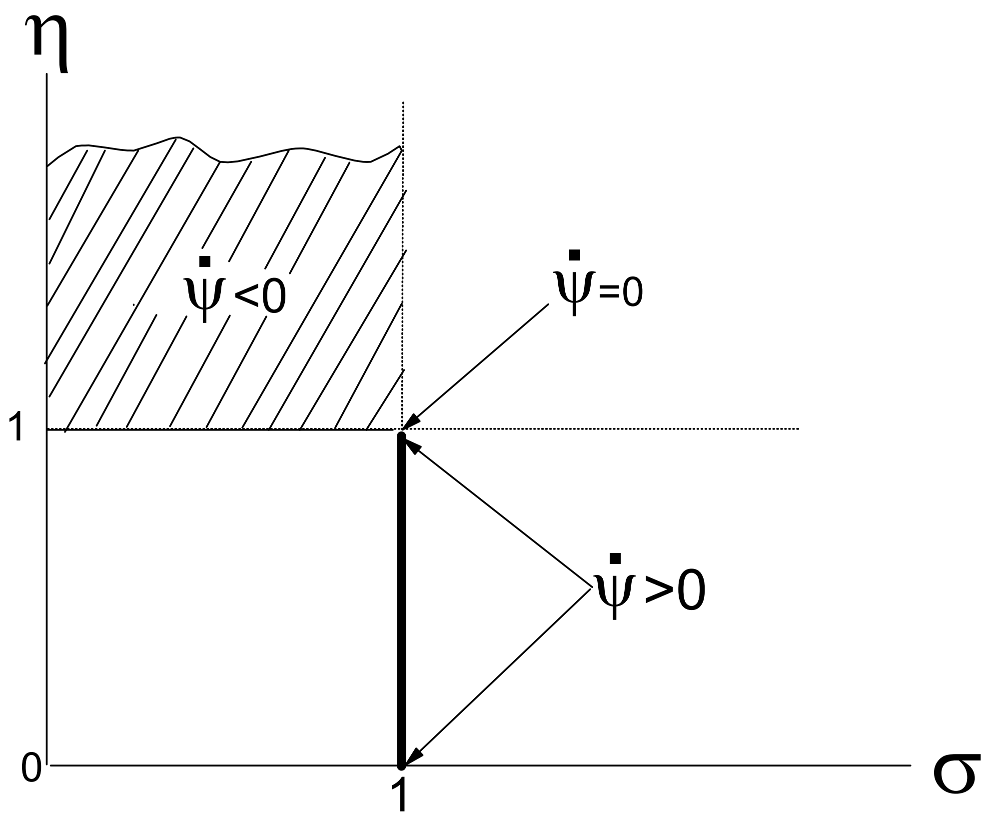

Figure 1