The Three-sector Growth Hypothesis and the Franklin-Euler-Malthus Exponential Economic growth model: Application to the analysis of relative GDP dynamics of Brazil, 1947-2007-2027.

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Abstract: The paper describes the interpolation and extrapolation of annual relative growth of GDP in the three basic economic activities: Primary, Secondary and Tertiary economic activities. The annual regional economic growth is evaluated with the help of the Franklin-Euler-Malthus Exponential growth model. As a regional example of annual economic growth in Brazil represents the results of interpolation and extrapolation of relative empirical dynamics of GDP in Brazil in the years 1947-2007-2027.

Key words: Franklin-Euler-Malthus Growth model, Interpolation/Extrapolation procedure. Three-sector growth hypothesis. Log-linear probabilistic chain of economic growth. GDP dynamics of Brazil.

I. Introduction.

Exponential (geometric) growth gives the simple straightforward model of growth.

Exponential Growth occurs when the growth rate γ for any growing in time socio-economic stock $f(\gamma, m)$ is directly proportional to the current value of stock. (Symbolically this means that $f(\gamma, m) = \gamma f(m)$). Examples of such stocks could be: distinct population or labor types, different built capital stocks (for instance, classified according to vintage), stocks of financial capital (currencies), types of economic output (production types), wealth (the General National Product GNP and General Domestic Product GDP) or any other economic, social, political types of quantities.

Historically the exponential growth of population, and economic production outputs attracted the most attention and opened the ways towards the construction of mathematical models of growth (Harris, 2001).

In this paper the classical population exponential growth model (Franklin, 1751; Euler, 1767; Malthus, 1798, 1826) is clarified and used for the evaluation (interpolation and extrapolation) of economic growth of modern production structure. The exponential growth model of the Franklin-Euler-Malthus became the methodological basis of the Three-sector Hypothesis of the economic growth .

Simon Kuznets stated in his Nobel lecture that “rapid changes in production structure are inevitable” (Kuznets, 1973, p. 250). In a recent survey presented by Jens Kruger, he argues that “the topic of structural change is frequently neglected in economic research (Kruger, 2008, p. 331). However, many authors devoted energy to analyzing the changing structure of the economy in the process of growth, as surveyed by Kruger. One of these is the Three-sector Growth hypothesis, which is an economic theory which divides economies into three aggregated sectors of activity: Primary activity (PR) including agriculture and extraction of raw materials, , Secondary activity (SD) including manufacturing , and Tertiary activities (TR) - services .

This division was initially developed by Clark (see three editions of his book "Conditions of Economic Progress", 1940, 1951, 1957) and Fourastie (see Fourastie, 1949).

Clark's studies spread widely over the economics, including the agricultural economics, macroeconomics, demography, economic policy, economic growth and national income accounting (see Maddison, 2004).He is credited with the introduction of the concept of Gross National Product (GNP) (at around the same time as Kuznets' introduction of Gross Domestic Product (GDP) (Kuznets , 1953)).

GNP gives a quantitative measure of total economic activity of a nation assessed yearly or quarterly. The GNP equals the GDP plus income earned by domestic residents through foreign investments minus the income earned by foreign investors in the domestic markets. GDP is calculated from the total value of goods and services produces in an economy over the specific period of time. Clark has stressed the dominance of different sectors economic activity at different stages of its development and modernization.

Clark introduced the analysis of comparative performance in three main sectors of economy and initiated the discussion (in *The Conditions of Economic Progress*, 1940, p 176) by quoting what he called the Petty's Law (Petty, 1676, see Hull' collection, 1899). "There is more to be gained by manufacture then husbandry, and by merchandise than manufacture." He stressed the significance of the three-sector breakdown and structural change in interpreting economic growth.

According to Fourastie the main focus of an economy's activity shifts from the primary activity, through the secondary activity and finally to tertiary sector. Forastie (1949) saw the process as essentially positive and writes on the increase in quality of life, social security, blossoming of education and culture, higher level of qualifications, humanization of work and avoidance of unemployment. Subsequent work by Kuznets (1966) elaborated upon these ideas, presenting a sectoral transitions perspective on macroeconomic growth and development.

In continuation of ideas of Petty, Clark, Forastie and Kuznets, in this paper an attempt will be made to measure the rates of change in agriculture, industry and services and present the analytical structure of interaction, development and dynamics of major three economic activities sectors. Let us formalize the Three-sector growth hypothesis: let $\gamma_{PR}, \gamma_{SD}, \gamma_{TR}$ be the average multiplicative rates of change for of these sectors; over time, (see chapter II). The three-sector growth hypothesis, including the Petty' Law means the following ordering: $\gamma_{TR} \geq \gamma_{SD} \geq 1 \geq \gamma_{PR}$. But obviously the different forms of such ordering exist for different countries in different time periods. This difference is caused by different forms of economic evolution in different countries.

In this paper we use as case study of economic growth of Brazil, 1947-2007.

In Brazil**,** from 1960 to 1980, one observes an increase in the share of services and parallel decreases in the share of agriculture, while the participation of industry first increased and then diminished. From 1980 to 1990, there was s significant grows in the share of services mainly at the expense of industry. After 1994, there was smaller increase in the share of services and a further reduction in the share of industry. (Da Fonceca, 2001)

As will be shown below the recent situation in Brazil corresponds to the inequalities $\gamma_{TR} > \gamma_{SD} > 1 > \gamma_{PR}$. This implies an increase of the GDP dynamics in Tertiary activity (Services) and Secondary activity (manufacturing).

In the next chapters the mathematical structure of dependences between rates of growth of three sectors will be presented in detail with the help of deterministic Franklin-Euler-Malthus growth model. Section II presents the analytical structure of Franklin-Euler-Malthus growth model as three-sector probabilistic chain giving the interpolation/extrapolation method of dynamics of sectoral change; next the approximate equation of the dependencies between the rates of growth of the sectoral activities will be presented. Section IV presents the probabilistic chain of GDP portions in Brazil, 1947-2007-2020.

II. The Probabilistic Three-sector GDP Franklin-Euler-Malthus Dynamics

Consider the three deterministic empirical sequences

$$\tilde{p}_t, \tilde{g}_t, \tilde{r}_t;\ \tilde{p}_t + \tilde{g}_t + \tilde{r} = 1;\ t = 0,1,...,T \qquad (1)$$

presenting the probabilistic dynamics of relative portions of annual GDP values generated by three major economic primary, secondary and tertiary activities during the time intervals $t = 0,1,...,T$.

The three-activity deterministic Franklin-Euler-Malthus dynamic Growth models based on classical population growth model (Franklin, 1751; Euler, 1767; Malthus, 1798, 1826; Hoppenstead, Peskin, 1992) and can be introduced in the form of the following iterative dynamic processes:

$$\begin{aligned} &p_{t+1} = \gamma_{PR} p_t\,, \quad q_{t+1} = \gamma_{SD} q_t\,, \quad r_{t+1} = \gamma_{TR} r_t \\ &t = 0,1,...,T, T+1, T+2,... \end{aligned} \qquad (2)$$

where p_t, q_t, r_t are the portions of annual GDP for Primary, Secondary and Tertiary economic activities; the growth parameters $\gamma_{PR}, \gamma_{SD}, \gamma_{TR}$ are the average annual growth rates and is the initial state of the iteration processes (2).

Let us have an economic interpretation of the model (2). Let us consider the absolute changes of the annual shares of GDP in the three economic activities:

$$\Delta p_t = p_{t+1} - p_t = (\gamma_{PR} - 1)\, p_t,$$
$$\Delta q_t = q_{t+1} - q_t = (\gamma_{SD} - 1)\, q_t, \qquad (3)$$
$$\Delta r_t = r_{t+1} - r_t = (\gamma_{TR} - 1)\, r_t$$

Obviously

$$\Delta p_t + \Delta q_t + \Delta r_t = 0 \qquad (4)$$

In the case of economy of Services:

$\gamma_{TR} > 1$

the following three qualitative sign distributions are existing:

Sign Δp_t	*Sign* Δq_t	*Sign* Δr_t	*Explanations*
−	+	+	$\gamma_{PR} < 1 < \gamma_{SD}, \gamma_{TR}$
−	−	+	$\gamma_{PR}, \gamma_{SD} < 1 < \gamma_{TR}$
+	−	+	$\gamma_{SD} < 1 < \gamma_{PR}, \gamma_{TR}$

Table 1. The qualitative descriptions of growth in the economics of Services,

Further it is easy to rewrite the three sector Franklin-Euler-Malthus model (2) in the simple form:

$$p_t = \gamma_{PR}^t p_0,\ q_t = \gamma_{SD}^t q_0,\ r_t = \gamma_{TR}^t r_0.$$
$$t = 0,1,...,T,T+1,T+2,... \qquad (5)$$

This form of the three-sector model presents the interpolation/extrapolation procedure for the empirical sequences (1) (interpolation for $t = 0,1,...,T;$; extrapolation for $t = T+1, T+2,...$

The probabilistic chain dynamics (2) is equivalent to the following logistic growth probabilistic chain (see Sonis, 1987, 2003):

$$p_{t+1} = \frac{\gamma_{PR} p_t}{\gamma_{PR} p_t + \gamma_{SD} q_t + \gamma_{TR} r_t},$$
$$q_{t+1} = \frac{\gamma_{SD} q_t}{\gamma_{PR} p_t + \gamma_{SD} q_t + \gamma_{TR} r_t}, \qquad (6)$$
$$r_{t+1} = \frac{\gamma_{TR} r_t}{\gamma_{PR} p_t + \gamma_{SD} q_t + \gamma_{TR} r_t}$$

The system of equations (5) is equivalent to the following system of difference equations:

$$
\begin{aligned}
p_{t+1} - p_t &= p_{t+1}\left[\left(1-\frac{\gamma_{SD}}{\gamma_{PR}}\right)q_t + \left(1-\frac{\gamma_{TR}}{\gamma_{PR}}\right)r_t\right],\\
q_{t+1} - q_t &= q_{t+1}\left[\left(1-\frac{\gamma_{PR}}{\gamma_{SD}}\right)p_t + \left(1-\frac{\gamma_{TR}}{\gamma_{SD}}\right)r_t\right],\\
r_{t+1} - r_t &= r_{t+1}\left[\left(1-\frac{\gamma_{PR}}{\gamma_{TR}}\right)p_t + \left(1-\frac{\gamma_{SD}}{\gamma_{TR}}\right)q_t\right]
\end{aligned}
\qquad (7)
$$

The expression $\frac{p_{t+1} - p_t}{p_{t+1}}$ presents the relative increment of the share p_t as a linear combination of the shares q_t, r_t. A similar interpretation of the second and third row in (7) can be made for the relative increments of the shares q_t, r_t. This provides the possibility for introducing the co-influence matrix:

$$
C = \begin{pmatrix} 0 & 1-\frac{\gamma_{SD}}{\gamma_{PR}} & 1-\frac{\gamma_{TR}}{\gamma_{PR}} \\ 1-\frac{\gamma_{PR}}{\gamma_{SD}} & 0 & 1-\frac{\gamma_{TR}}{\gamma_{SD}} \\ 1-\frac{\gamma_{PR}}{\gamma_{TR}} & 1-\frac{\gamma_{SD}}{\gamma_{TR}} & 0 \end{pmatrix} \qquad (8)
$$

presenting the indirect influence of the shares of growth of different sectors of activity on the relative increments of the shares of other sectors. The sign matrix Sign C adds some information to the relative growth of portions of GDP in regional economics. For the case of economics of services of the type $\gamma_{PR} < 1 < \gamma_{SD}, \gamma_{TR}$ the form of Sign matrix $SignC = \begin{pmatrix} 0 & - & - \\ + & 0 & - \\ + & + & 0 \end{pmatrix}$ is.

Using the dependence (5) of the relative portions p_t, q_t, r_t of GDP on rates of growth $\gamma_{PR}, \gamma_{SD}, \gamma_{TR}$ and on initial distribution p_0, q_0, r_0 of GDP one obtains the following probabilistic chain

$$
\begin{aligned}
p_t &= \frac{p_0\gamma_{PR}^t}{p_0\gamma^t + q_0\gamma_{SD}^t + r_0\,\gamma_{TR}^t}; \\
q_t &= \frac{q_0\gamma_{SD}^t}{p_0\gamma^t + q_0\gamma_{SD}^t + r_0\,\gamma_{TR}^t}; \\
r_t &= \frac{r_0\,\gamma_{TR}^t}{p_0\gamma^t + q_0\gamma_{SD}^t + r_0\,\gamma_{TR}^t}
\end{aligned} \tag{9}
$$

An approximation procedure for the derivation of the annual growth parameters $\gamma_{PR}, \gamma_{SD}, \gamma_{TR}$ now will be proposed as

$$
\gamma_{PR} = \frac{1}{T}\sum_{i=1}^{T}\frac{p_i}{p_{i-1}}; \gamma_{SD} = \frac{1}{T}\sum_{i=1}^{T}\frac{q_i}{q_{i-1}}; \gamma_{TR} = \frac{1}{T}\sum_{i=1}^{T}\frac{r_i}{r_{i-1}} \tag{10}
$$

Using the formulae (10) we obtain the following approximate equation

$$
\frac{1}{3}\left(\gamma_{APR} + \gamma_{SD} + \gamma_{TR}\right) \approx 1 \tag{11}
$$

which represents the connection between the rates of growth of all sector rates of growth. For the proof of the approximate equation (11) we replace the Arithmetical mean with the Geometrical mean according to formula:

$$
\frac{1}{3}(a+b+c) \approx (abc)^{\frac{1}{3}} \tag{12}
$$

We will have the following expression

$$\frac{1}{3}(\gamma_{APR}+\gamma_{SD}+\gamma_{TR})=$$

$$=\frac{1}{3}(\frac{1}{T}\sum_{i=1}^{T}\frac{p_i}{p_{i-1}}+\frac{1}{T}\sum_{i=1}^{T}\frac{q_i}{q_{i-1}}+\frac{1}{T}\sum_{i=1}^{T}\frac{r_i}{r_{i-1}})=$$

$$=\frac{1}{T}\sum_{i=1}^{T}\frac{1}{3}(\frac{p_i}{p_{i-1}}+\frac{q_i}{q_{i-1}}+\frac{r_i}{r_{i-1}})=\frac{1}{T}\sum_{i=1}^{T}\frac{1}{3}(\frac{p_i}{p_{i-1}}+\frac{q_i}{q_{i-1}}+\frac{r_i}{r_{i-1}})\approx$$

$$\approx\frac{1}{T}\sum_{i=1}^{T}(\frac{p_i}{p_{i-1}}\frac{q_i}{q_{i-1}}\frac{r_i}{r_{i-1}})^{1/3}=\frac{1}{T}\sum_{i=1}^{T}\frac{(p_iq_ir_i)^{1/3}}{(p_{i-1}q_{i-1}r_{i-1})^{1/3}}\approx$$

$$\approx\frac{1}{T}\sum_{i=1}^{T}\frac{\frac{1}{3}(p_i+q_i+r_i)}{\frac{1}{3}(p_{i-1}+q_{i-1}+r_{i-1})^{1/3}}=1$$

The approximated initial state p_0, q_0, r_0 of the Franklin-Euler-Malthus model is given by the help of averages:

$$p_0=\frac{P}{P+Q+R};\ q_0=\frac{Q}{P+Q+R};\ r_0=\frac{R}{P+Q+R} \qquad (13)$$

where

$$P=\frac{1}{T+1}\sum_{i=0}^{T}\frac{p_i}{\gamma_{PR}^{i}},\ Q=\frac{1}{T+1}\sum_{i=0}^{T}\frac{q_i}{\gamma_{SD}^{i}},\ R=\frac{1}{T+1}\sum_{i=0}^{T}\frac{r_i}{\gamma_{TP}^{i}} \qquad (14)$$

The partial sectoral correlation coefficients R_{PR}, R_{SD}, R_{TR} representing the goodness of fit between the empirical GDP dynamics (1) and the model (5) can be calculated with the help of equations:

$$R_{PR} = \frac{\sum_{t=0}^{T} \tilde{\tilde{p}}_t \, p_t}{\left\{ \sum_{t=0}^{T} \left(\tilde{p}_t \right)^2 \right\}^{\frac{1}{2}} \left\{ \sum_{t=0}^{T} \left(p_t \right)^2 \right\}^{\frac{1}{2}}} \;;$$

$$R_{SD} = \frac{\sum_{t=0}^{T} \tilde{q}_t \, q_t}{\left\{ \sum_{t=0}^{T} \left(\tilde{q}_t \right)^2 \right\}^{\frac{1}{2}} \left\{ \sum_{t=0}^{T} \left(q_t \right)^2 \right\}^{\frac{1}{2}}} \;; \qquad (15)$$

$$R_{TR} = \frac{\sum_{t=0}^{T} \tilde{r}_t \, r_t}{\left\{ \sum_{t=0}^{T} \left(\tilde{r}_t \right)^2 \right\}^{\frac{1}{2}} \left\{ \sum_{t=0}^{T} \left(r_t \right)^2 \right\}^{\frac{1}{2}}}$$

III. Brazilian Relative GDP growth dynamics of Primary, Secondary and Tertiary economic activities, Brazil, 1947-2007-2027.

Brazil is a good case to apply the above methodology, since it is a large country, and has been under intensive productive changes, notably after 1990. Starting from a typical Third World agricultural economy previously to World War II, in which coffee exports accounted for most of its foreign-oriented activities, the country followed an import substitution strategy until the mid-80s, but remained quite closed to foreign trade. Starting in 1990, an opening-up process took place, and nowadays, although still relatively closed, its economy is more open. Coinciding with this opening process, a strong movement of increased agricultural production took place, together with an impressive investment in energy-related crops, of which its ethanol program is emblematic. As a result, Brazil is a major international player nowadays in beef, grains, cotton, coffee, and, of course, energy-related products. These changes took place in a context of a fast-growing internal market, since its population increased from 41 millions in 1940 to 187 millions in 2008.

These productive changes had consequences in the shares of the three main productive sectors in total output. In the decade 1947-1957 the shares of primary and secondary activities were almost the same, around 20% - 25%, with the tertiary sector accounting for the remaining share. From

1958 to the1993, the share of the secondary activities changes between 30% and 40. This came about from a decrease in the share of primary activities, reaching around 9%- 10% in late 1987s, and even the tertiary activities, with around 50% in that period. Opening the economy to foreign trade cased important changes in these shares. Primary activities, in spite of the success of Brazilian exports in the above mentioned products, stabilized their share around 5%-6% in 1993-2007.[1].

Given the changes mentioned, we will deal in this paper with a period, 1947-2007. Table 2 represents the distribution of Brazilian GDP during, 1947-2007.

<<Insert table 2 here>>

These empirical statistical data can be used for the evaluation of parameters of Franklin-Euler-Malthus models (5, 6). Equation (10) provides the aggregate relative growth rates: $\gamma_{TR} = 1.003679 : \gamma_{SD} = 1.003095;\ \gamma_{PR} = 0.981764$. The type of the GDP growth in Brazil has a form:

$$\gamma_{TR} > \gamma_{SD} > 1 > \gamma_{PR} \qquad (16)$$

with average total rate of growth

$$\Delta = \frac{1}{3}(\gamma_{TR} + \gamma_{SD} + \gamma_{PR}) = 0.996187 \approx 1 \qquad (17)$$

This means that the aggregate relative change in Tertiary and Secondary Activities is larger than in Primary activities. Because a sectorial rate of growth is bigger then 1 the GDP is increasing in these activities. The rate of growth in Primary activity is less 1, so the portion of GDP in Primary activity is decreasing.

The initial states of GDP dynamics calculated with the help of equation (13) are:

$$p_0 = 0.171552,\ q_0 = 0.312588,\ r_0 = 0.51546 \quad .$$

The interpolation and extrapolation GDP dynamics provide the interpolation and extrapolation forecast of Brazilian GDP relative dynamics for 1947-2007-2027 (see table 2):

[1] See Contas Regionalis de Brazil, 1947-2007, and Guilhoto and Hewings (2001) and Baer (2007) for a detailed description of the evolution of Brazilian economy.

The goodness of fit between empirical GDP Primary relative dynamics and the model can be measured with the help of the partial correlation coefficient (15), which generates the following value $R_{PR} = 0.987311$. Using the same equation, the goodness of fit between empirical GDP Secondary relative dynamics is $R_{SD} = 0.9855925$. For Tertiary relative dynamics the value partial correlation coefficient is $R_{TR} = 0.994496$.

Figure 1 presents the geometrical presentation of the GDP empirical and model dynamics for the three macro sectors in Brazil including interpolation for 1947-2007 and extrapolation for 2008 through 2027 (*cf.* table 2)

<<insert figure 1 here>>

The co-influence matrix (8) based on the parameters of GDP growth of economic activities $\gamma_{PR} = 0.981764;\ \gamma_{SD} = 1.003095; \gamma_{TR} = 1.003679$ has the following form

$$C = \begin{pmatrix} 0 & 1-\frac{\gamma_{SD}}{\gamma_{PR}} & 1-\frac{\gamma_{TR}}{\gamma_{PR}} \\ 1-\frac{\gamma_{PM}}{\gamma_{SD}} & 0 & 1-\frac{\gamma_{TR}}{\gamma_{SD}} \\ 1-\frac{\gamma_{PM}}{\gamma_{SD}} & 1-\frac{\gamma_{SD}}{\gamma_{TR}} & 0 \end{pmatrix} = \begin{pmatrix} 0 & -0.02173 & -0.02232 \\ 0.02185 & 0 & -0.00058 \\ 0.02127 & 0.00058 & 0 \end{pmatrix} \qquad (18)$$

This co-influence matrix generates the chain of the probabilistic distribution of GDP for the aggregated sectors in Brazil (see figure 1). These chains reveal the growth of the relative portion (%) of GDP in services, and manufacturing, and the decrease in relative portion (%) of GDP in agriculture and mineral industry.

IV. Conclusions

The theoretical part of this paper introduced consideration of the Franklin-Euler-Malthus dynamic growth model in the form of the log-linear probabilistic growth chain. This provides the basis for the Three-sector Hypothesis in the form of interpolation/extrapolation procedure of relative probabilistic non-linear growth dynamics.

The second part of the paper presents the application of the interpolation/extrapolation technique to the analysis of the three-sector growth of GDP in Brazil, based on empirical data of annual GDP in Primary, Secondary and Tertiary activities for the period 1947-2007, interpolation of this data using average annual rates of GDP growth and the extrapolation of Brazilian economic dynamics for 2008-2027.

It is possible to stress that the analytical scheme of GDP dynamics and especially the formula (16) presented in this paper can be expanded to the analysis of GDP of multi-sectoral and multuregional dynamics in Brazil and in this way to classify types of regional growth of GDP in Brazil in different time periods. Here the notions of so called Matrioska principle (Matrioshka principle represent the nesting hierarchy of economic regions) (*cp* Sonis and Hewings, 1990; Sonis, Hewings and Okuyama, 2001) could be used.

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PR model	SD model	TR model	PR	SD	TR	Years
0.171952	0.312588	0.51546	0.20739	0.252086	0.540524	1947
0.168862	0.313641	0.517497	0.227835	0.241753	0.530412	1948
0.165817	0.314676	0.519507	0.235765	0.247331	0.516904	1949
0.162816	0.315694	0.521491	0.242619	0.241484	0.515897	1950
0.159858	0.316694	0.523448	0.237713	0.251314	0.510974	1951
0.156944	0.317677	0.525379	0.249869	0.241758	0.508373	1952
0.154074	0.318643	0.527283	0.235564	0.253937	0.510499	1953
0.151247	0.319592	0.529161	0.241168	0.257622	0.50121	1954
0.148462	0.320524	0.531014	0.234654	0.256427	0.508919	1955
0.14572	0.32144	0.53284	0.210891	0.273109	0.516	1956
0.14302	0.322338	0.534641	0.204277	0.278089	0.517634	1957
0.140362	0.323221	0.536417	0.183997	0.311161	0.504842	1958
0.137745	0.324087	0.538168	0.171594	0.329832	0.498574	1959
0.13517	0.324937	0.539893	0.177569	0.322362	0.500069	1960
0.132635	0.325771	0.541594	0.169637	0.325328	0.505035	1961
0.13014	0.326589	0.54327	0.174645	0.324778	0.500577	1962
0.127686	0.327392	0.544922	0.159475	0.330964	0.509561	1963
0.125271	0.328179	0.54655	0.162751	0.325158	0.512091	1964
0.122895	0.32895	0.548154	0.158609	0.319553	0.521838	1965
0.120559	0.329707	0.549735	0.141475	0.327608	0.530917	1966
0.11826	0.330448	0.551292	0.137131	0.32034	0.54253	1967
0.116	0.331175	0.552825	0.11787	0.34767	0.53446	1968
0.113777	0.331886	0.554336	0.11394	0.352445	0.533615	1969
0.111592	0.332584	0.555824	0.115515	0.358393	0.526091	1970
0.109443	0.333267	0.55729	0.121693	0.362193	0.516114	1971
0.107331	0.333936	0.558734	0.122524	0.369934	0.507542	1972
0.105254	0.33459	0.560155	0.119233	0.395891	0.484877	1973
0.103213	0.335231	0.561555	0.114382	0.404928	0.48069	1974
0.101208	0.335859	0.562934	0.107469	0.403718	0.488813	1975
0.099236	0.336473	0.564291	0.108584	0.399055	0.492361	1976
0.097299	0.337073	0.565627	0.126075	0.386387	0.487538	1977
0.095396	0.337661	0.566943	0.102598	0.394947	0.502455	1978
0.093527	0.338235	0.568238	0.099073	0.400527	0.5004	1979
0.09169	0.338797	0.569513	0.101126	0.409347	0.489527	1980
0.089885	0.339346	0.570768	0.101124	0.400423	0.498453	1981
0.088113	0.339883	0.572004	0.087263	0.412079	0.500658	1982
0.086372	0.340407	0.57322	0.109458	0.389261	0.501281	1983
0.084663	0.34092	0.574417	0.121541	0.407257	0.471201	1984
0.082984	0.34142	0.575595	0.111165	0.422704	0.466131	1985
0.081336	0.341909	0.576755	0.111953	0.437095	0.450952	1986
0.079717	0.342386	0.577896	0.093315	0.409862	0.496823	1987
0.078128	0.342852	0.57902	0.097509	0.400378	0.502113	1988
0.076568	0.343307	0.580125	0.077429	0.366357	0.556214	1989
0.075037	0.34375	0.581213	0.069128	0.330301	0.60057	1990

0.073534	0.344183	0.582283	0.068994	0.320369	0.610637	1991
0.072058	0.344605	0.583337	0.062272	0.312293	0.625435	1992
0.07061	0.345017	0.584373	0.057728	0.317669	0.624603	1993
0.069189	0.345418	0.585393	0.05772	0.296463	0.645817	1994
0.067794	0.345809	0.586397	0.057712	0.275258	0.66703	1995
0.066426	0.34619	0.587385	0.055136	0.259839	0.685024	1996
0.065083	0.346561	0.588357	0.053964	0.261288	0.684748	1997
0.063765	0.346922	0.589313	0.055248	0.256583	0.688169	1998
0.062472	0.347274	0.590254	0.054732	0.259458	0.685811	1999
0.061204	0.347616	0.59118	0.056028	0.277318	0.666654	2000
0.05996	0.347949	0.592091	0.059734	0.269236	0.67103	2001
0.05874	0.348273	0.592987	0.066176	0.270519	0.663304	2002
0.057543	0.348588	0.593869	0.07386	0.278458	0.647683	2003
0.056368	0.348895	0.594737	0.069133	0.301136	0.62973	2004
0.055217	0.349192	0.595591	0.057084	0.292748	0.650168	2005
0.054087	0.349481	0.596431	0.051576	0.301195	0.647229	2006
0.05298	0.349762	0.597258	0.055158	0.287057	0.657785	2007
0.051894	0.350035	0.598072				2008
0.050828	0.350299	0.598872				2009
0.049784	0.350556	0.59966				2010
0.04876	0.350805	0.600435				2011
0.047756	0.351046	0.601198				2012
0.046771	0.35128	0.601948				2013
0.045806	0.351507	0.602687				2014
0.04486	0.351726	0.603414				2015
0.043933	0.351938	0.604129				2016
0.043024	0.352143	0.604833				2017
0.042132	0.352341	0.605526				2018
0.041259	0.352533	0.606208				2019
0.040403	0.352718	0.60688				2020
0.039564	0.352896	0.60754				2021
0.038741	0.353068	0.60819				2022
0.037935	0.353234	0.608831				2023
0.037145	0.353394	0.609461				2024
0.036371	0.353548	0.610081				2025
0.035613	0.353696	0.610691				2026
0.034869	0.353838	0.611293				2027

Sourse: Contas Regionalis de Brazil, 1947-2007**.**

Table 2. Brazilian relative sectoral GDP growth dynamics of Primary, Secondary and Tertiary economic activities, Brazil: Empirical data and Interpolation 1947-2007; Extrapolation 2008-2027.

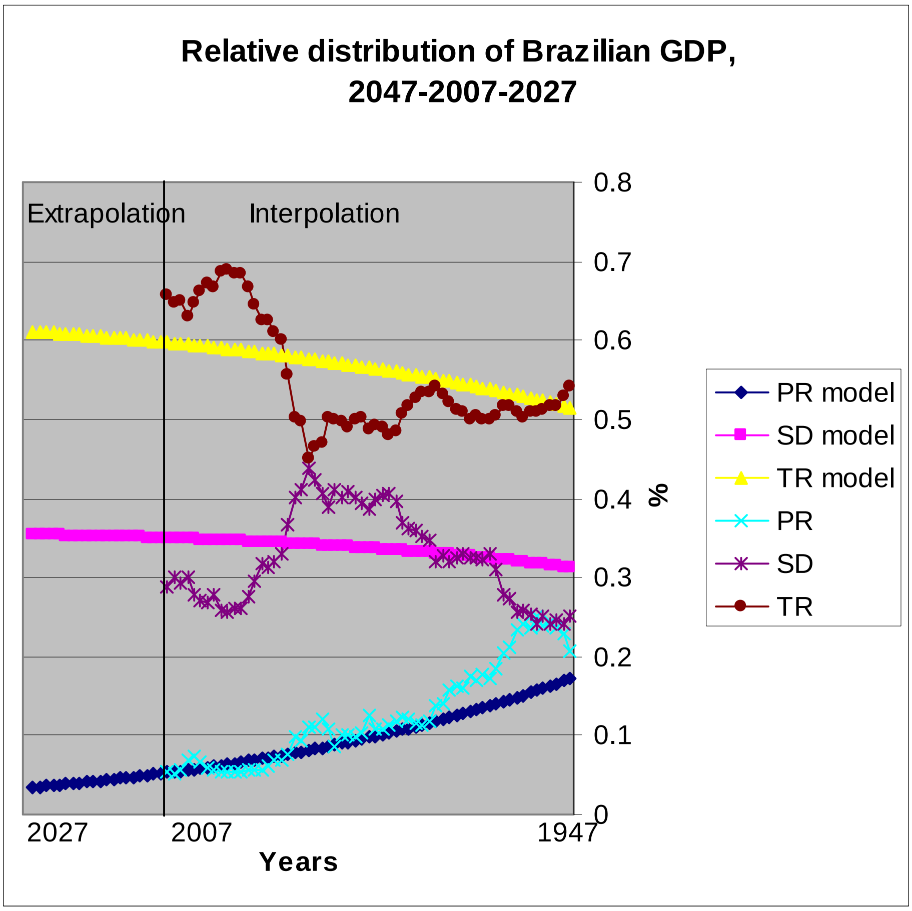

Figure 1. Interpolation and Extrapolation of Brazil GDP dynamics in different sectors and total annual GDP dynamics.